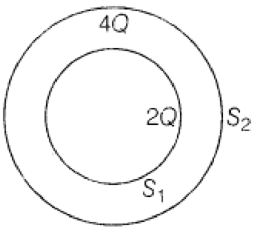
# Gauss’s Law

## 1 Mark Questions

1.Consider two hollow concentric spheres $S_1$ and $S_2$ enclosing charges 20 and 40 respectively, as shown in the figure, (i) Find out the ratio of the electric flux through them, (ii) How will the electric flux through the spheres $S_1$ change if a medium of dielectric constant $e_r$ is introduced in the space inside $S_1$ in place of air? Deduce the necessary expression. [All India 2014]




Ans.

(*i*) According to Gauss’ theorem,

$$\phi = \frac{\Sigma q}{\varepsilon_0 \varepsilon} \propto \Sigma q$$

$$\frac{\phi_{S_1}}{\phi_{S_2}} = \frac{2Q}{2Q + 4Q} = \frac{1}{3}$$

(*ii*) If the medium is filled in $S_1$, then

$$\phi_{S_1} = \frac{\Sigma q}{\varepsilon_0 \varepsilon_r} = \frac{2Q}{\varepsilon_0 \varepsilon_r}$$

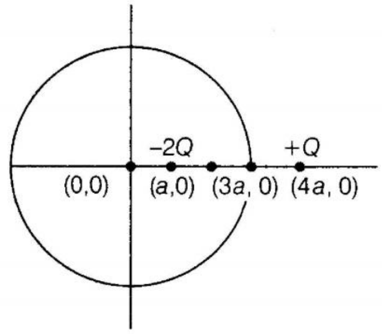
2.Two charges of magnitudes –20 and + O are located at points (a, 0) and (4a, 0), respectively. What is the electric flux due to these charges through a sphere of radius 3a with its centre at the origin? [All India 2013]

Ans.

Gauss' theorem states that the total electric flux linked with closed surface $S$ is

$$\phi_E = \oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{E_0}$$

where, $q$ is the total charge enclosed by the closed Gaussian (imaginary) surface.




The sphere enclose charge = –2Q

Therefore, $\phi = \frac{2Q}{\varepsilon_0}$ (inwards) **(1)**

3.A charge q is placed at the centre of a cube of side L. What is the electric flux passing through each face of the cube? [All India 2010; Foreign 2010]

Ans.

By Gauss' theorem, total electric flux linked with a closed surface is given by

$$\phi = \frac{q}{\varepsilon_0}$$

where, $q$ is the total charge enclosed by the closed surface.

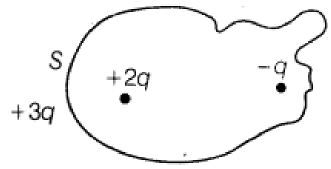
∴ Total electric flux linked with cube, $\phi = \frac{q}{\varepsilon_0}$

As charge is at centre, therefore, electric flux is symmetrically distributed through all 6 faces.

Flux linked with each face $= \frac{1}{6}\phi$

$$= \frac{1}{6} \times \frac{q}{\varepsilon_0} = \frac{q}{6\,\varepsilon_0}$$ **(1)**

4.Figure shows three point charges,+ 2q, – q and + 3q. Two charges + 2q and – q are enclosed within a surface S. What is the electric flux due to this configuration through the surface S? [Delhi 2010]




Ans.

Electric flux through the closed surface $S$ is

$$\phi_S = \frac{\Sigma q}{\varepsilon_0}$$

$$= \frac{+2q - q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad \phi_S = \frac{q}{\varepsilon_0} \quad \textbf{(1)}$$

Charge $+3q$ is outside the closed surface $S$, therefore, it would not be taken into consideration in applying Gauss' theorem.

5.If the radius of the Gaussian surface enclosing a charge is halved, how does the electric flux through the Gaussian surface change?[All India 2009,2008]

**Ans**.Total charge enclosed by the Gaussian surface remains same even when radius is halved. Therefore, total electric flux remains constant as per Gauss' theorem. There will not be any change in electric flux through the Gaussian surface.

## 2 Marks Questions

6.Given a uniform electric held E =5 x $10^3$ i N/C, find the flux of this held through a square of 10 cm on a side whose plane is parallel to the YZ-plane. What would be the flux through the same square if the plane makes an angle of 30° with the X-axis? [Delhi 2014]

Ans.

Given, electric field intensity

$\mathbf{E} = 5 \times 10^3\ \hat{\mathbf{i}}$ N/C

Magnitude of electric field intensity

$|\mathbf{E}| = 5 \times 10^3$ N/C

Side of square, $S$ = 10 cm = 0.1 m

Area of square, $A = (0.1)^2 = 0.01\ m^2$

The plane of the square is parallel to the YZ-plane. **(1)**

Hence, the angle between the unit vector normal to the plane and electric field is zero.

i.e., $\theta = 0°$

∴ Flux through the plane,

$$\phi = |\mathbf{E}| \times A \cos\theta$$

$$\phi = 5 \times 10^3 \times 0.01 \cos 0°$$

$$\phi = 50\ \text{N-m}^2/\text{C} \quad \textbf{(1)}$$

If the plane makes an angle of 30° with the x-axis, then $\theta = 60°$

∴ Flux through the plane,

$$\phi = |\mathbf{E}| \times A \times \cos 60°$$

$$= 5 \times 10^3 \times 0.01 \times \cos 60°$$

$$= 25\ \text{N - m}^2/\text{C}$$

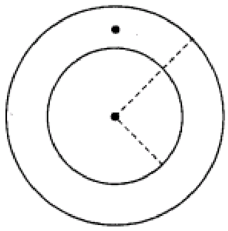
7.Given a uniform electric held E =2 x $10^3$ i N/C, find the flux of this held through a square of side 20 cm, whose plane is parahel to the YZ-plane. What would be the flux through the same square if the plane makes an angle of 30° with the X-axis? [Delhi 2014, HOTS]

**Ans.** Refer to ans. 6. (Ans 40 $Nm^2/C$)

8.Given a uniform electric held E = 4 x $10^3$ i N/C. Find the flux of this field through a square of 5 cm on a side whose plane is parallel to the YZ-plane. What would be the flux through the same square if the plane makes an angle of 30° with the X-axis? [Delhi 2014, HOTS]

**Ans**.Refer to ans 6. (Ans. 5 $Nm^2/C$)

9.A sphere $S_t$ of radius q enclosed a net charge Q. If there is another concentric sphere $S_2$ of radius $r_{2(}r_2$ > q) enclosing charge 20, hnd the ratio of the electric hux through $S_1$ and $S_2$. How will the electric flux through sphere Sj change if a medium of dielectric constant K is introduced in the space inside $S_2$ in place Of air? [All India 2014]



Ans.

According to Gauss' law,

Flux through $S_1$,

$$\phi_1 = \frac{Q}{\varepsilon_0} \quad \text{...(i)}$$

**(1/2)**

Flux through $S_2$,

$$\phi_2 = \frac{Q+2Q}{\varepsilon_0} = \frac{3Q}{\varepsilon_0} \quad \text{...(ii)}$$

Ratio of flues = $\frac{\phi_1}{\phi_2}$

From Eqs. (i) and (ii), we get

$$= \frac{Q}{\varepsilon_0} \times \frac{\varepsilon_0}{3Q} = \frac{1}{3}$$

**(1)**

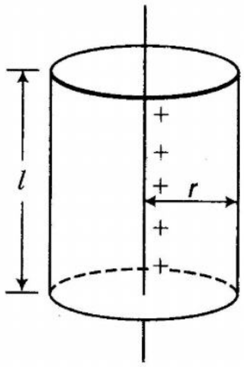
There is no change in the flux through $S_1$ with dielectric medium inside the sphere $S_2$. **(1/2)**

**10.** A thin straight infinitely long conduction wire having charge density $\lambda$ is enclosed by a cylindrical surface of radius $r$ and length $l$, its axis coinciding with the length of the wire. Find the expression for the electric flux through the surface of the cylinder. **[All India 2011]**

**Ans.** A thin straight conducting wire will be a uniform linear charge distribution.

Let q charge be enclosed by the cylindrical surface.

∴ Linear charge density, $\lambda = \frac{q}{l}$




Charge enclosed by the cylindrical surface

∴ $$q = \lambda l \quad \text{...(i)}$$ **(1/2)**

By Gauss' theorem,

∴ Total electric flux through the surface of cylinder

$$\phi = \frac{q}{\varepsilon_0} \quad \text{[Gauss' theorem]}$$ **(1)**

∴ $$\phi = \frac{\lambda l}{\varepsilon_0} \quad \text{[From Eq. (i)]}$$ **(1/2)**

11.Two charged conducting spheres of radii $r_t$ and $r_2$ connected to each other by a wire. Find the ratio of electric fields at the surfaces of the two spheres. [Delhi 2011 c]

Ans.

> When two charged conducting spheres are connected, then charge flows between the two till their potentials become same.

Electric potential on the surface of connected charged conducting spheres would be equal.

i.e. $$V_1 = V_2$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

[Assuming $q_1$ and $q_2$ are charges on the spheres connected to each other and $r_1, r_2$ are their radii.]

$$\frac{q_1}{r_1} = \frac{q_2}{r_2} \Rightarrow \frac{q_1}{q_2} = \frac{r_1}{r_2} \quad \text{...(i)}$$ **(1)**

Now, ratio of electric field intensities

$$\frac{E_1}{E_2} = \frac{\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r_1^2}}{\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{r_2^2}} = \frac{q_1}{q_2} \times \frac{r_2^2}{r_1^2}$$

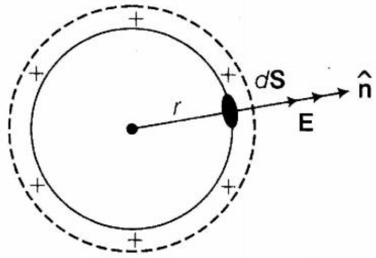
$$\frac{E_1}{E_2} = \left(\frac{q_1}{q_2}\right) \times \frac{r_2^2}{r_1^2} = \frac{r_1}{r_2} \times \frac{r_2^2}{r_1^2} \quad \text{[From Eq. (i)]}$$

$$\frac{E_1}{E_2} = \frac{r_2}{r_1}$$ **(1)**

**12.** Show that the electric field at the surface of a charged conductor is given by $E = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{n}}$, where $\sigma$ is the surface charge density and $\hat{\mathbf{n}}$ is a unit vector normal to the surface in the outward direction. **[All India 2010]**

Ans.

Let $q$ charge be uniformly distributed over the spherical shell of radius $r$.




$\therefore$ Surface charge density on spherical shell

$$\sigma = \frac{q}{4\pi r^2} \quad \text{...(i)}$$

**(1/2)**

$\because$ Electric field intensity on the surface of spherical shell

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\hat{\mathbf{n}}$$

**(1/2)**

[$\because$ **E** acts along radially outward and along $\hat{\mathbf{n}}$]

$$\mathbf{E} = \frac{\left(\frac{q}{4\pi r^2}\right)}{\varepsilon_0}\hat{\mathbf{n}}, \Rightarrow \mathbf{E} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{n}} \quad \text{...(ii)}$$

**(1)**

**13.A spherical conducting shell of inner radius $R_1$ and outer radius $R_2$ has a charge Q. A charge q is placed at the centre of the shell. [All India 2010c]**

**(i)What is the surface charge density on the (a) inner surface, (b) outer surface of the shell?**

**(ii)Write the expression for the electric field at a point to $x>R_2$ from the centre of the shell.**

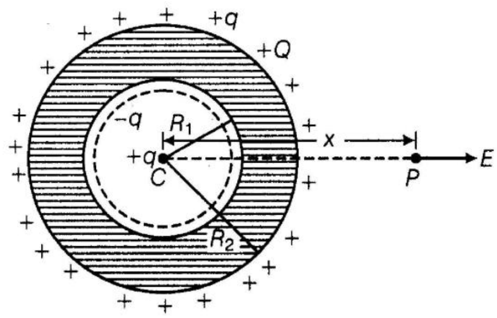
**Ans**.Here, two points are important

(i) Charge resides on the outer surface of spherical conductor (skin effect).

(ii) Equal charge of opposite nature induces in the surface of conductor nearer to source charge.

(i) (a) Charge produced on inner surface due to induction = $-q$

∴ Surface charge density of inner surface = $\frac{-q}{4\pi R_1^2}$




When charge $-q$ is induced on inner walls, then equal charge $+q$ is produced at outer surface.

(b) Charge on outer surface = $q + Q$

∴ Surface charge density of outer surface

$$= \frac{q+Q}{4\pi R_2^2}$$ **(1)**

(ii) Electric field intensity at point $P$ separated by a distance $x (x > R_2)$

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{(q+Q)}{x^2}$$

[along $CP$ and away from spherical shell] **(1)**

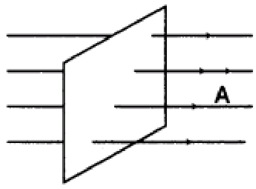
Whole charge is assumed to be concentrated at the centre.

**14.Define electric flux. Write its SI unit. A charge gis enclosed by a spherical surface of radius [All India 2009]**

**Ans.** The total electric flux linked with a surface is equal to the total number of electric lines of force passing through the surface when surface is held normal to the direction of electric field.

$\phi = EA$

If surface is placed in non-uniform, electric fild then electric field.




Total electric flux linked with the closed surface

$$\phi = \oint_S \mathbf{E} \cdot d\mathbf{S}$$

The SI unit of electric flux is N-m$^2$/C. **(2)**

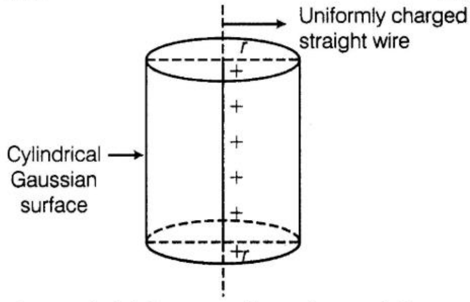
**15.Draw the shapes of the suitable Gaussian surfaces while applying Gauss' law to calculate the electric field due to**

**(i)a uniformly charged long straight wire.**

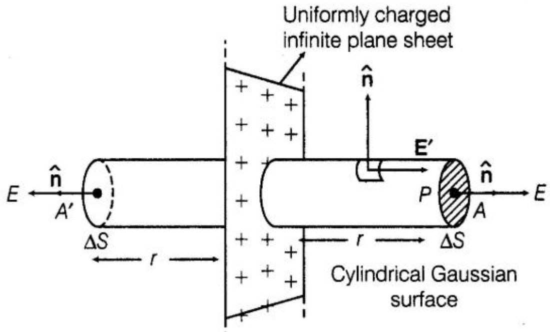
**(ii)a uniformly charged infinite plane sheet. [Delhi 2009 C]**

**Ans.** The surface that we choose for application of Gauss' theorem is called Gaussian surface. We usually choose a spherical Gaussian surface.

(*i*) Electric field due to a long straight wire of sheet **(1)**




(*ii*) Electric field due to a plane sheet of charge




**16.** A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of $80.0\,\mu C/m^2$

(*i*) Find the charge on the sphere.

(*ii*) What is the total electric flux leaving the surface of the sphere?

**[Delhi 2009C]**

Ans.

(*i*) Radius of sphere, $R = \frac{d}{2} = \frac{2.4}{2} = 1.2$ m

Surface charge density,

$$\sigma = 80 \times 10^{-6}\ C/m^2$$

$$\therefore \quad \sigma = \frac{q}{4\pi R^2}$$

or $\quad q = 4\pi R^2 \sigma$

Charge on the sphere

$$q = 4 \times 3.14 \times (1.2)^2 \times 80 \times 10^{-6}\ C$$

$$q = 1.447 \times 10^{-3}\ C$$

(*ii*) According to Gauss' theorem,

Electric flux, $\phi = \frac{q}{\varepsilon_0}$

$$\phi = \frac{1.447 \times 10^{-3}}{8.85 \times 10^{-12}}\ Nm^2/C$$

$$\phi = 1.63 \times 10^{8}\ Nm^2/C$$

**17.** Two large parallel thin metallic plates are placed close to each other. The plates have surface charge densities of opposite signs and of magnitude $2.0 \times 10^{-12} C/m^2$. Calculatethe electric field intensity

(*i*) in the outer region of the plates

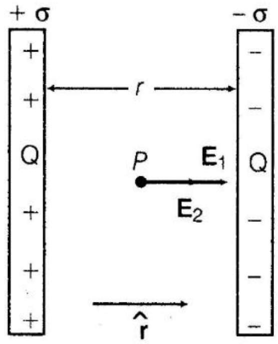
(*ii*) in the interior region between the plates. **[Foreign 2008]**

Ans.

Let $\hat{\mathbf{r}}$ be a unit vector directed from left to right.

Let *P* and *Q* are two points in the outer and inner region of two plates, respectively.

Charge densities on plates are $+\sigma$ and $-\sigma$.




$\therefore \quad \sigma = 2 \times 10^{-12}$ C/m$^2$

(*i*) Electric field at point *P* in the outer region of the plates

$$\mathbf{E}_1 = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} \text{ or } \mathbf{E}_2 = -\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$$

$\therefore$ Net field in the outer region of the plates (i.e., at *P*)

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} - \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} = 0 \qquad \textbf{(1)}$$

(*ii*) Electric field at point *Q* in the interior of two plates

$$\mathbf{E}_1 = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$$

$$\mathbf{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$$

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} + \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{r}}$$

$\mathbf{E} = \frac{\sigma}{\varepsilon_0}$ [From positive plate to negative plate]

But, $\quad \sigma = 2 \times 10^{-2}$ C/m$^2$

$$\therefore \quad E = \frac{2 \times 10^{-12}}{8.85 \times 10^{-12}}$$

$$E = 2.25 \times 10^{-1} \text{ N/C} \qquad \textbf{(1)}$$

**18.** A point charge causes and electric flux $-3\times10^{-14}$ $Nm^2/C$ to pass through a spherical Gaussian surface.

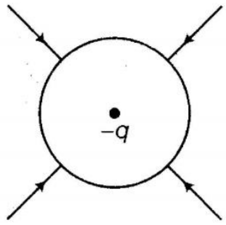
(*i*) Calculate the value of the point charge.

(*ii*) If the radius of the Gaussian surface is double, how much flux would pass through the surface?

**[Foreign 2008]**

Ans.

(*i*) By Gauss' theorem, total electric flux through closed Gaussian surface is given by




$$\phi = \frac{q}{\varepsilon_0}$$

$\therefore$ $$q = \phi\varepsilon_0$$

But, electric flux passing through the surface

$$\phi = -3\times10^{-14} Nm^2/C$$

$\therefore$ $$q = (-3\times10^{-14})\times8.85\times10^{-12}$$

$$= -26.55\times10^{-26} C$$

$$q = -2.655\times10^{-25} C$$ **(1)**

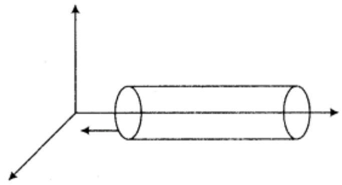
(*ii*) Electric flux passing through the surface remains unchanged because it depends only on charge enclosed by the surface and is independent of its size. **(1)**

## 3 Marks Questions

19.A hollow cylindrical box of lenght 1 m and area of cross-section 25 $cm^2$ is placed in a three-dimensional coordinate system as shown in the figure.

The electric field in the region is given by $E = 50\,x\hat{\mathbf{i}}$, where $E$ is in $NC^{-1}$ and x is in metre.

Find



(*i*) net flux through the cylinder.

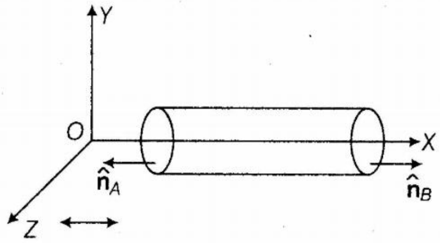
(*ii*) charge enclosed by the cylinder.

**[Delhi 2013]**

Ans.

(*i*) Given, $E = 50 \times i$

and $\Delta S = 25 \text{ cm}^2 = 25 \times 10^{-4} \text{ m}^2$




As the electric field is only along the *X*-axis, so flux will pass only through the cross-section of the cylinder.

Magnitude of electric field at cross-section *A*,

$$E_A = 50 \times 1 = 50 \text{ NC}^{-1}$$

Magnitude of electric field at cross-section *B*,

$$E_B = 50 \times 2 = 50 \text{ NC}^{-1}$$ **(1/2)**

The corresponding electric fluxes are

$$\phi_A = \mathbf{E}_A \cdot \Delta\mathbf{S} = 50 \times 25 \times 10^{-4} \times \cos 180°$$
$$= -0.125 \text{ Nm}^2\text{C}^{-1}$$

$$\phi_B = \mathbf{E}_B \cdot \Delta\mathbf{S} = 100 \times 25 \times 10^{-4} \times \cos 0°$$
$$= 0.25 \text{ Nm}^2\text{C}^{-1}$$

So, the net flux through the cylinder, **(1)**

$$\phi = \phi_A + \phi_B$$
$$= 0.125 + 0.25 = 0.125 \text{ Nm}^2\text{C}^{-1}$$

(*ii*) Using Gauss' law,

$$\oint \mathbf{E} \cdot d\mathbf{l} = \frac{q}{\varepsilon_0}$$

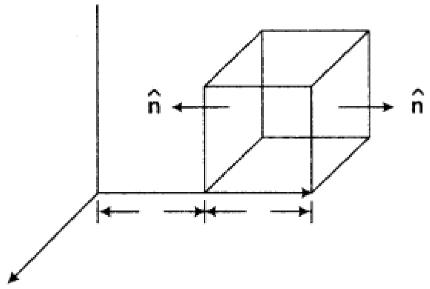
$$\Rightarrow \quad 0.125 = \frac{q}{8.85 \times 10^{-12}}$$

$$\Rightarrow \quad q = 8.85 \times 0.125 \times 10^{-12}$$

$$q = 1.1 \times 10^{-12} \text{ C}$$ **(1/2)**

So, the charge enclosed by the cylinder is $1.1 \times 10^{-12}$ C. **(1)**

20.State Gauss' law in electrostatics. A cube which each side a is kept in an electric field given by E = as shown in the figure, where C is a positive dimensional constant. Find out




(i)the electric flux through the cube

(ii)the net charge inside the cube.[Foreign 2012]

Ans.

Gauss' law states that the total electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times, the magnitude of the charge enclosed by it is.

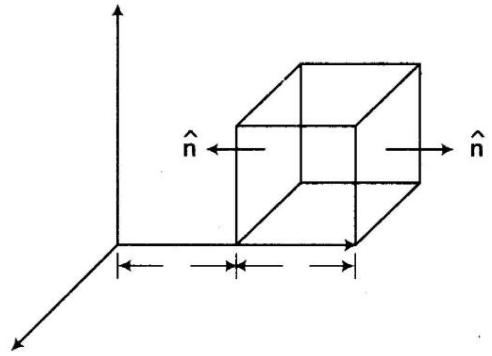
$$\phi = \frac{q}{\varepsilon_0}$$

Here, $\varepsilon_0$ is the absolute permittivity of the free space and $q$ is the total charge enclosed.

Also, $\phi = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$

where, $\mathbf{E}$ is the electric field at the area element $d\mathbf{S}$.

Now, the electric field $\mathbf{E} = Cx\hat{\mathbf{i}}$ is in $X$-direction only. So, faces with surface normal vector perpendicular to this field would give zero electric flux, i.e. $\phi = E\,dS \cos 90° = 0$ through it.



So, flux would be across only two surfaces.

Magnitude of $E$ at left face,

$$E_L = Cx = Ca \qquad [x = a \text{ at left face}]$$

Magnitude of $E$ at right face

$$E_R = Cx$$
$$= C2a = 2aC \qquad [x = 2a \text{ at right face}]$$

Thus, corresponding fluxes are

$$\phi_L = \mathbf{E}_L \cdot d\mathbf{S} = E_L \, dS \cos\theta$$
$$= - aC \times a^2 \qquad [\text{As}, \theta = 180°]$$

$$\phi_R = \mathbf{E}_R \cdot d\mathbf{S} = 2aC \, dS \cos\theta \qquad [\because \theta = 0°]$$
$$= 2aCa^2$$
$$= 2a^3C$$ **(1)**

(*i*) Now, net flux through the cube is

$$= \phi_L + \phi_R$$
$$= - a^3C + 2a^3C$$
$$= a^3C \text{ Nm}^2\text{C}^{-1}$$ **(1)**

(*ii*) Net charge inside the cube

Again, we can use Gauss' law to find total charge $q$ inside the cube.

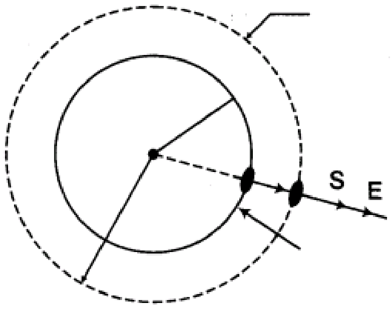
We have $\phi = \dfrac{q}{\varepsilon_0}$

or $q = \phi \varepsilon_0$

$$q = a^3C\varepsilon_0 \text{ coulomb}$$ **(1)**

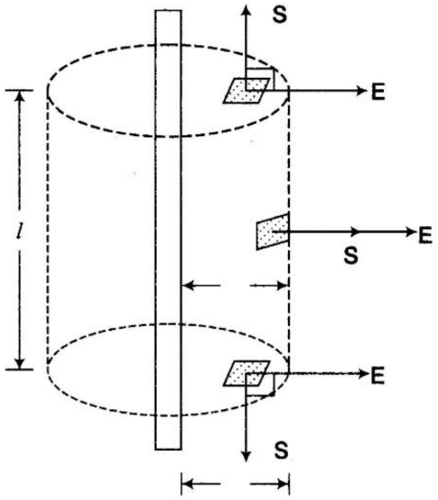
**21.Using Gauss' law, obtain the expression for the electric field due to uniformly charged spherical shell of radius R at a point outside the shell. Draw a graph showing the variation of electric field with r, for r > R and r < R.[All India 2011]**

**Ans**.Let us consider charge +q is uniformly distributed over a spherical shell of radius R. Let £ is to be obtained at P lying outside of spherical shell.




Eat any point is radially outward (if charge q is positive) and has same magnitude at all points which lie at the same distance r from centre of spherical shell such that r> R.

Therefore, Gaussian surface is concentric sphere of radius $r$ such that $r > R$. **(1/2)**




Cylindrical Gaussian surface for line charge

Since, Gaussian surface enclosed charge $q$ inside it.

By Gauss' theorem,

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0} \Rightarrow \oint E\,dS\,\cos 0° = \frac{q}{\varepsilon_0}$$

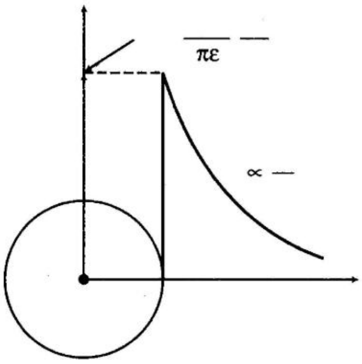
[$\because$ **E** and $d\mathbf{S}$ are along the same direction]

$$E \oint dS = \frac{q}{\varepsilon_0}$$

[$\because$ Magnitude of $E$ is same at every point on Gaussian surface]

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \quad \textbf{(1)}$$

Now, graph



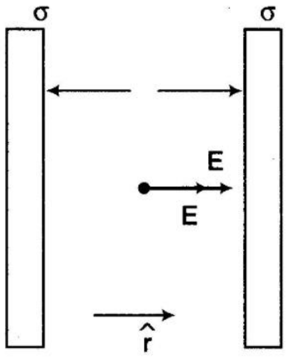

Variation of $E$ with $r$ for a spherical shell of charge **(1)**

**22.** Use Gauss' law to derive the expression for the electric field between two uniformly charge parallel sheets with surface charge densities $\sigma$ and $-\sigma$, respectively.

**[All India 2009]**

Ans.

Let us consider two uniformly charge, large parallel sheets carrying charge densities $+\sigma$ and $-\sigma$ respectively, are separated by a small distance from each other.




By Gauss' law, it can be proved that electric field intensity due to a uniformly charged infinite plane sheet at any nearby is given

$$E = \frac{\sigma}{2\varepsilon_0} \quad \ldots(i)$$

**(1)**

The electric field is directed normally outward from the plane sheet if nature of charge on sheet is positive and normally inward if charge is of negative nature.

Let $\hat{\mathbf{r}}$ represents unit vector directed from positive plate to negative plate.

Now, Electric Field Intensity (EFI) at any point $P$ between the two plates is given by

(*i*) $\mathbf{E}_1 = +\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$ [Due to positive plate]

(*ii*) $\mathbf{E}_2 = +\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$ [Due to negative plate]

**(1)**

∴ Electric field intensity at $P$ point is given by

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

$$= \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} + \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$$

$$\mathbf{E} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{r}}$$

Thus, a uniform electric field is produced between the two infinite parallel plane sheet of charge which is directed from positive plate to negative plate. **(1)**

23.State Gauss' law in electrostatics. Using this law, derive an expression for the electric field due to a uniformly charged infinite plane Sheet. [Delhi 2009]

Ans.

Gauss' law states that the total electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times, the magnitude of the charge enclosed by it is.

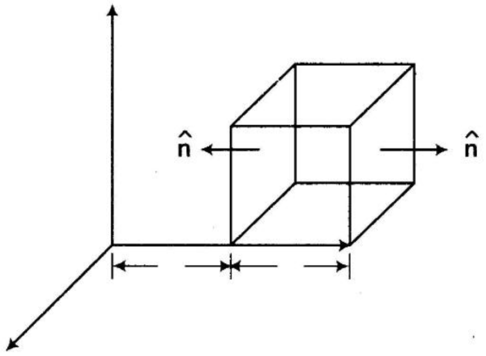
$$\phi = \frac{q}{\varepsilon_0}$$

Here, $\varepsilon_0$ is the absolute permittivity of the free space and $q$ is the total charge enclosed.

Also, $$\phi = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

where, $\mathbf{E}$ is the electric field at the area element $d\mathbf{S}$.

Now, the electric field $\mathbf{E} = Cx\hat{\mathbf{i}}$ is in $X$-direction only. So, faces with surface normal vector perpendicular to this field would give zero electric flux, i.e. $\phi = E\,dS \cos 90° = 0$ through it.



So, flux would be across only two surfaces.

Magnitude of $E$ at left face,

$$E_L = Cx = Ca \qquad [x = a \text{ at left face}]$$

Magnitude of $E$ at right face

$$E_R = Cx$$
$$= C2a = 2aC \qquad [x = 2a \text{ at right face}]$$

Thus, corresponding fluxes are

$$\phi_L = \mathbf{E}_L \cdot d\mathbf{S} = E_L \, dS \cos\theta$$
$$= -aC \times a^2 \qquad [\text{As, } \theta = 180°]$$

$$\phi_R = \mathbf{E}_R \cdot d\mathbf{S} = 2aC \, dS \cos\theta \qquad [\because \theta = 0°]$$
$$= 2aCa^2$$
$$= 2a^3C$$
**(1)**

(*i*) Now, net flux through the cube is

$$= \phi_L + \phi_R$$
$$= -a^3C + 2a^3C$$
$$= a^3C \text{ Nm}^2\text{C}^{-1}$$
**(1)**

(*ii*) Net charge inside the cube

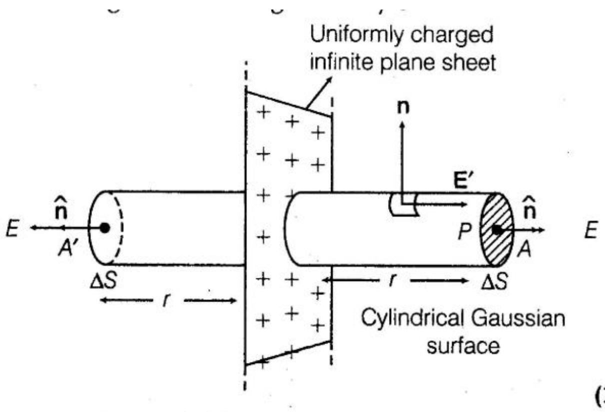
Again, we can use Gauss' law to find total charge $q$ inside the cube.

We have $\phi = \frac{q}{\varepsilon_0}$

or $q = \phi \varepsilon_0$

$$q = a^3C\varepsilon_0 \text{ coulomb}$$
**(1)**

Let us consider a large plane sheet of charge having surface charge density sigma




**(1)**

Let electric field is to be obtained at a point $P$ at a distance $r$ from it. It is obvious that Gaussian surface will be a cylinder of cross-sectional area $A$ and length $2r$ with its axis perpendicular to plane sheet of charge.

Now, applying Gauss' law over the closed Gaussian surface.

$$\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$ **(1)**

$$\int_{CSA} \mathbf{E} \cdot d\mathbf{S} + \int_{CSA} \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

$$\int_{CSA} E\, dS \cos 0° + \int_{CSA} E\, dS \cos 90° = \frac{q}{\varepsilon_0}$$

[As **E** and $d$**S** are along the same direction by at CSA **E** perpendicular to $d$**S**]

$$\int_{CSA} dS = \frac{q}{\varepsilon_0} \Rightarrow E \times 2A = \frac{q}{\varepsilon_0} \quad \text{...(i)}$$

$$E = \frac{q}{2A\varepsilon_0} = \frac{\sigma}{2\varepsilon_0} \quad \text{[From Eq. (i)]}$$

$\therefore$ Electric field intensity $= \dfrac{\sigma}{2\varepsilon_0}$

The direction of $E$ is normal to the plane sheet and directed away from sheet when charge on plate is positive and *vice-versa*. **(1)**

Closed cylinder comprises of two caps and Curved Surface Area (CSA).

**24.** State Gauss' law in electrostatics. Using this law derive an expression for the electric field due to a long straight wire of linear charge density λ C/m. **[Delhi 2009]**

Ans. (i)

Gauss' law states that the total electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times, the magnitude of the charge enclosed by it is.

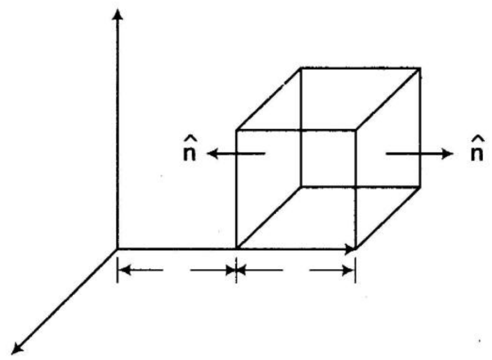
$$\phi = \frac{q}{\varepsilon_0}$$

Here, $\varepsilon_0$ is the absolute permittivity of the free space and $q$ is the total charge enclosed.

Also, $\phi = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$

where, **E** is the electric field at the area element $d\mathbf{S}$.

Now, the electric field $\mathbf{E} = Cx\hat{\mathbf{i}}$ is in $X$-direction only. So, faces with surface normal vector perpendicular to this field would give zero electric flux, i.e. $\phi = E\,dS \cos 90° = 0$ through it.



So, flux would be across only two surfaces.
Magnitude of $E$ at left face,

$$E_L = Cx = Ca \qquad [x = a \text{ at left face}]$$

Magnitude of $E$ at right face

$$E_R = Cx$$
$$= C2a = 2aC \qquad [x = 2a \text{ at right face}]$$

Thus, corresponding fluxes are

$$\phi_L = \mathbf{E}_L \cdot d\mathbf{S} = E_L \, dS \, \cos\theta$$
$$= -aC \times a^2 \qquad [\text{As, } \theta = 180°]$$
$$\phi_R = \mathbf{E}_R \cdot d\mathbf{S} = 2aC \, dS \, \cos\theta \qquad [\because \theta = 0°]$$
$$= 2aCa^2$$
$$= 2a^3C$$

**(1)**

(*i*) Now, net flux through the cube is

$$= \phi_L + \phi_R$$
$$= -a^3C + 2a^3C$$
$$= a^3C \text{ Nm}^2\text{C}^{-1}$$

**(1)**

(*ii*) Net charge inside the cube
Again, we can use Gauss' law to find total charge $q$ inside the cube.

We have $\phi = \dfrac{q}{\varepsilon_0}$

or $q = \phi\,\varepsilon_0$

$$q = a^3C\varepsilon_0 \text{ coulomb}$$

**(1)**

(*ii*) Let us consider a long straight wire carrying $+q$ charge on its length $l$ and linear charge density $\lambda$ C/m.

$$\because \quad \lambda = \frac{q}{l}$$

$$\Rightarrow \quad q = \lambda l \quad \text{...(i)}$$

Let electric field intensity is to be obtained at a distance $r$ from it. Since, magnitude of **E** due to long charged wire is same at every point which lie at the same distance from the wire. So, Gaussian surface will be a cylinder of radius $r$ and length $l$ such that wire lies along the axis as shown in figure.

**(1)**

$\because$ Angle between **E** and $d$**S** is 90° at caps, whereas 0° at any point on curved surface of a cylinder.

Now, applying Gauss' theorem

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0} \quad \textbf{(1)}$$

$$\int_{CSA} \mathbf{E} \cdot d\mathbf{S} + \int_{CSA} \mathbf{E} \cdot d\mathbf{S} = \frac{\lambda}{\varepsilon_0} \quad \text{[From Eq. (i)]}$$

[CSA = Close Surface Area]

$$\int_{CSA} E\, dS \cos 90° + \int_{CSA} E\, dS \cos 0° = \frac{\lambda}{\varepsilon_0}$$

($S_1$ and $S_3$ are caps and $S_2$ represents CSA)

$$0 + \int_{CSA} E\, dS = \frac{\lambda l}{\varepsilon_0} \quad [\because \cos 90° = 0]$$

$$E \int_{CSA} dS = \frac{\lambda l}{\varepsilon_0}$$

[$\because E$ is a constant at every point at CSA]

$$E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0} \quad \left[\because \int_{CSA} dS = 2\pi r l\right]$$

$$E = \frac{\lambda l}{2\pi\varepsilon_0 r l}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \textbf{(1)}$$

## 4 Marks Questions

25.Using Gauss' law, deduce the expression for the electric field due to a uniformly charged spherical conducting shell of radius R at a point

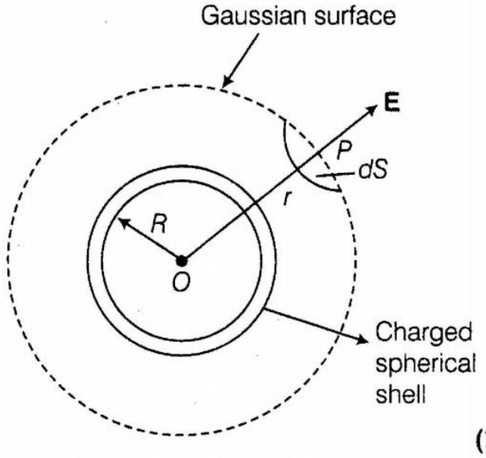
(i)outside the shell

(ii)inside the shell

Plot a graph showing variation of electric field as a function of r > R and r < R.{r being the distance from the centre of the shell) [All India 2013]

Ans.

Electric field due to a uniformly charged thin spherical shell.




**(1)**

(*i*) **When point *P* lies outside the spherical shell**

Suppose that we have to calculate electric field at the point $P$ at a distance $r\,(r > R)$ from its centre. Draw the Gaussian surface through point $P$, so as to enclose the charged spherical shell. The Gaussian surface is a spherical shell of radius $r$ and centre $O$.

Let $\mathbf{E}$ be the electric field at point $P$. Then, the electric flux through area element $d\mathbf{S}$ is given by

$$d\phi = \mathbf{E} \cdot d\mathbf{S} \qquad \textbf{(1)}$$

Since, $d\mathbf{S}$ also along normal to the surface.

$$d\phi = E\,dS$$

$\therefore$ Total electric flux through the Gaussian surface is given by

$$\phi = \oint_S E\,dS = E\oint_S dS$$

Now, $\oint dS = 4\pi r^2$

$$\therefore \qquad \phi = E \times 4\pi r^2 \qquad \text{...(i)}$$

Since, the charge enclosed by the Gaussian surface is $q$. According to Gauss' theorem.

$$\phi = \frac{q}{\varepsilon_0} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we obtain

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \qquad [\text{for } r > R]$$ **(1)**

(*ii*) **When point *P* lies inside the spherical shell**

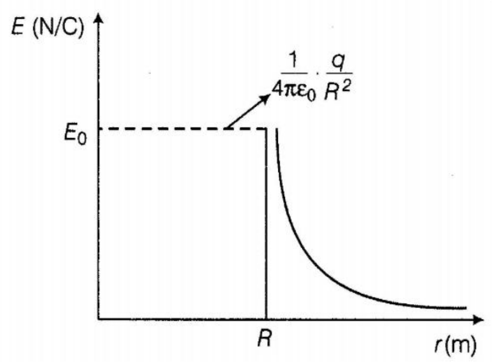
In such a case, the Gaussian surface enclosed no charge.

According to Gauss law,

$$E \times 4\pi r^2 = 0$$

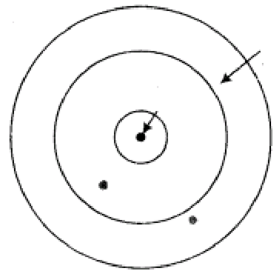
*i.e.*, $E = 0$ [for $r < R$]

**Graph showing the variation of electric field as a function of *r*** **(2)**




26.(i) Define electric flux. Write its SI unit.

(ii) A small metal sphere carrying charge + Q is located at the centre of a spherical cavity inside a large uncharged metallic spherical shell as shown in the figure.Use Gauss' law to find the expressions for the electric field at points $P_l$ and $P_2$.



(iii)Draw the pattern of electric field lines in this arrangement.[Delhi 2012 C]

Ans.

(*i*) **Electric flux** Electric flux over an area in an electric field represents the total number of electric lines of force crossing the area in a direction normal to the plane of the area. The SI unit of electric flux is N-m$^2$/C **(1)**

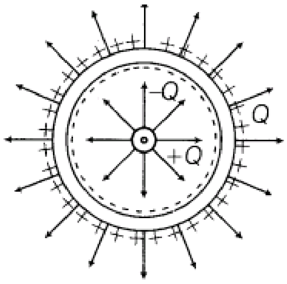
(*ii*) Using Gauss' theorem,

$$E \times 4\pi r_1^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r_1^2}$$

Field at point $P_2 = 0$, because the electric field inside the conductor is zero. **(1)**

(iii) The electric field lines due to arrangement is shown as below:




Charges will be uniformly distributed on all the surfaces hence, all field lines will be uniformly separated.

**27.Define electric flux. Write its SI unit, (ii) Using Gauss' law, prove that the electric field at a point due to a uniformly charged infinite plane sheet is independent of distance from it.**
**How is the field directed if**
**(a)the sheet is positively charged**
**(b)negatively charged? [Delhi 2012]**

**Ans**.(i) Electric flux Electric flux over an area in an electric field represents the total number of electric lines of force crossing the area in a direction normal to the plane of the area. The SI unit of electric flux is N-m$^2$/C

(ii)

Gauss' law states that the total electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times, the magnitude of the charge enclosed by it is.

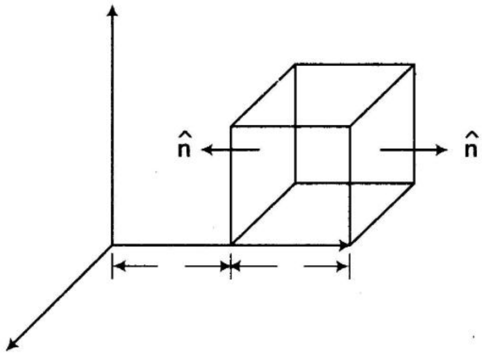
$$\phi = \frac{q}{\varepsilon_0}$$

Here, $\varepsilon_0$ is the absolute permittivity of the free space and $q$ is the total charge enclosed.

Also, $$\phi = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

where, **E** is the electric field at the area element $d$**S**.

Now, the electric field $\mathbf{E} = Cx\hat{\mathbf{i}}$ is in $X$-direction only. So, faces with surface normal vector perpendicular to this field would give zero electric flux, i.e. $\phi = E\,dS\cos 90° = 0$ through it.



So, flux would be across only two surfaces.
Magnitude of $E$ at left face,

$$E_L = Cx = Ca \qquad [x = a \text{ at left face}]$$

Magnitude of $E$ at right face

$$E_R = Cx$$
$$= C2a = 2aC \qquad [x = 2a \text{ at right face}]$$

Thus, corresponding fluxes are

$$\phi_L = \mathbf{E}_L \cdot d\mathbf{S} = E_L\, dS \cos\theta$$
$$= -aC \times a^2 \qquad [\text{As, } \theta = 180°]$$

$$\phi_R = \mathbf{E}_R \cdot d\mathbf{S} = 2aC\, dS \cos\theta \qquad [\because \theta = 0°]$$
$$= 2aCa^2$$
$$= 2a^3C$$ **(1)**

(*i*) Now, net flux through the cube is

$$= \phi_L + \phi_R$$
$$= -a^3C + 2a^3C$$
$$= a^3C \text{ Nm}^2\text{C}^{-1}$$ **(1)**

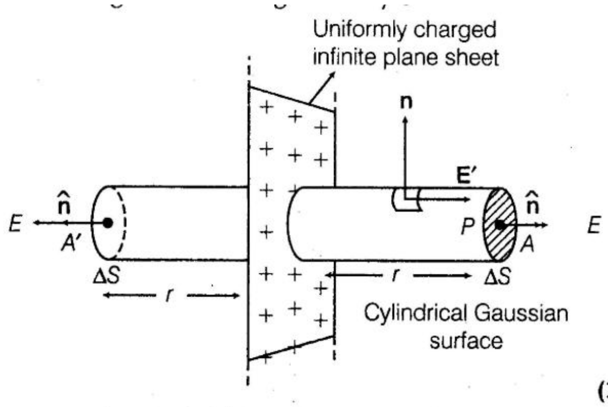
(*ii*) Net charge inside the cube
Again, we can use Gauss' law to find total charge $q$ inside the cube.

We have $\phi = \dfrac{q}{\varepsilon_0}$

or $q = \phi\,\varepsilon_0$

$$q = a^3C\varepsilon_0 \text{ coulomb}$$ **(1)**

Let us consider a large plane sheet of charge having surface charge density sigma




**(1)**

Let electric field is to be obtained at a point $P$ at a distance $r$ from it. It is obvious that Gaussian surface will be a cylinder of cross-sectional area $A$ and length $2r$ with its axis perpendicular to plane sheet of charge.

Now, applying Gauss' law over the closed Gaussian surface.

$$\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$ **(1)**

$$\int_{CSA} \mathbf{E} \cdot d\mathbf{S} + \int_{CSA} \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

$$\int_{CSA} E\, dS \cos 0° + \int_{CSA} E\, dS \cos 90° = \frac{q}{\varepsilon_0}$$

[As **E** and $d\mathbf{S}$ are along the same direction by at CSA **E** perpendicular to $d\mathbf{S}$]

$$\int_{CSA} dS = \frac{q}{\varepsilon_0} \Rightarrow E \times 2A = \frac{q}{\varepsilon_0} \quad \text{...(i)}$$

$$E = \frac{q}{2A\varepsilon_0} = \frac{\sigma}{2\varepsilon_0} \quad \text{[From Eq. (i)]}$$

$\therefore$ Electric field intensity $= \dfrac{\sigma}{2\varepsilon_0}$

The direction of $E$ is normal to the plane sheet and directed away from sheet when charge on plate is positive and *vice-versa*. **(1)**

Closed cylinder comprises of two caps and Curved Surface Area (CSA).

The field directed

- Normally away from the sheet when sheet is positively charged.
- Normally inward towards the sheet when plane sheet is negatively charged.

28.(i) State Gauss' law. Use it to deduce the expression for the electric field due to a uniformly charged thin spherical shell at points

- inside the shell and
- outside the shell.

(ii) Two identical metallic spheres A and B having charges +40 and – 100 are kept a certain distance apart. A third identical uncharged sphere C is first placed in contact with sphere A and then with sphere B. Then, spheres A and Bare brought in contact and then separated. Find the charges on the spheres A and B.[All India 2011C]

Ans.(i)

Gauss' law states that the total electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times, the magnitude of the charge enclosed by it is.

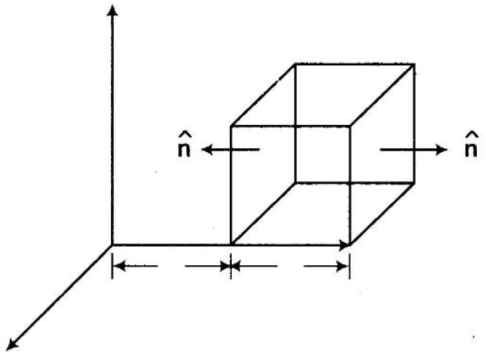
$$\phi = \frac{q}{\varepsilon_0}$$

Here, $\varepsilon_0$ is the absolute permittivity of the free space and $q$ is the total charge enclosed.

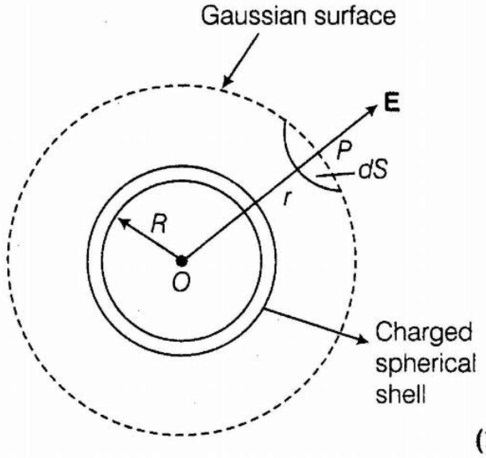
Also, $$\phi = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

where, $\mathbf{E}$ is the electric field at the area element $d\mathbf{S}$.

Now, the electric field $\mathbf{E} = Cx\hat{\mathbf{i}}$ is in $X$-direction only. So, faces with surface normal vector perpendicular to this field would give zero electric flux, i.e. $\phi = E\,dS \cos 90° = 0$ through it.



Electric field due to a uniformly charged thin spherical shell.




**(1)**

(*i*) **When point *P* lies outside the spherical shell**

Suppose that we have to calculate electric field at the point $P$ at a distance $r\,(r > R)$ from its centre. Draw the Gaussian surface through point $P$, so as to enclose the charged spherical shell. The Gaussian surface is a spherical shell of radius $r$ and centre $O$.

Let $\mathbf{E}$ be the electric field at point $P$. Then, the electric flux through area element $d\mathbf{S}$ is given by

$$d\phi = \mathbf{E} \cdot d\mathbf{S} \qquad \textbf{(1)}$$

Since, $d\mathbf{S}$ also along normal to the surface.

$$d\phi = E\, dS$$

∴ Total electric flux through the Gaussian surface is given by

$$\phi = \oint_S E\, dS = E \oint_S dS$$

Now, $\oint dS = 4\pi r^2$

$$\therefore \qquad \phi = E \times 4\pi r^2 \qquad \text{...(i)}$$

Since, the charge enclosed by the Gaussian surface is $q$. According to Gauss' theorem.

$$\phi = \frac{q}{\varepsilon_0} \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we obtain

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \qquad [\text{for } r > R]$$ **(1)**

(*ii*) **When point *P* lies inside the spherical shell**

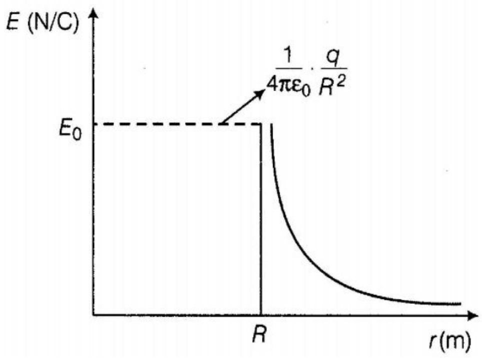
In such a case, the Gaussian surface enclosed no charge.

According to Gauss law,

$$E \times 4\pi r^2 = 0$$

*i.e.*, $E = 0$ $\quad [\text{for } r < R]$

**Graph showing the variation of electric field as a function of *r*** **(2)**




(ii)

Gauss' law states that the total electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times, the magnitude of the charge enclosed by it is.

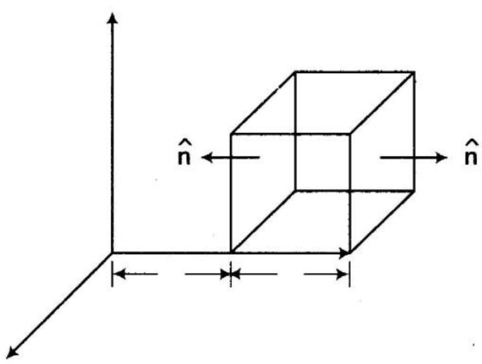
$$\phi = \frac{q}{\varepsilon_0}$$

Here, $\varepsilon_0$ is the absolute permittivity of the free space and $q$ is the total charge enclosed.

Also, $\phi = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$

where, **E** is the electric field at the area element $d\mathbf{S}$.

Now, the electric field $\mathbf{E} = Cx\hat{\mathbf{i}}$ is in $X$-direction only. So, faces with surface normal vector perpendicular to this field would give zero electric flux, i.e. $\phi = E\, dS \cos 90° = 0$ through it.



So, flux would be across only two surfaces.
Magnitude of $E$ at left face,

$$E_L = Cx = Ca \qquad [x = a \text{ at left face}]$$

Magnitude of $E$ at right face

$$E_R = Cx$$
$$= C2a = 2aC \qquad [x = 2a \text{ at right face}]$$

Thus, corresponding fluxes are

$$\phi_L = \mathbf{E}_L \cdot d\mathbf{S} = E_L\, dS \cos\theta$$
$$= -aC \times a^2 \qquad [\text{As, } \theta = 180°]$$
$$\phi_R = \mathbf{E}_R \cdot d\mathbf{S} = 2aC\, dS \cos\theta \qquad [\because \theta = 0°]$$
$$= 2aCa^2$$
$$= 2a^3C \qquad \textbf{(1)}$$

(*i*) Now, net flux through the cube is

$$= \phi_L + \phi_R$$
$$= -a^3C + 2a^3C$$
$$= a^3C \text{ Nm}^2\text{C}^{-1} \qquad \textbf{(1)}$$

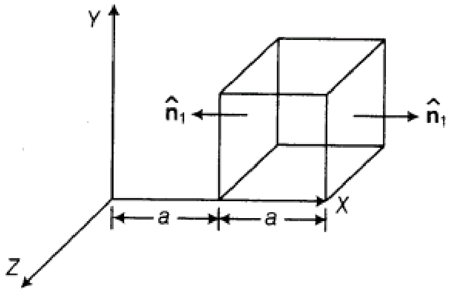
(*ii*) Net charge inside the cube
Again, we can use Gauss' law to find total charge $q$ inside the cube.

We have $\phi = \dfrac{q}{\varepsilon_0}$

or $q = \phi\,\varepsilon_0$

$$q = a^3C\varepsilon_0 \text{ coulomb} \qquad \textbf{(1)}$$

29.(i) Define electric flux. Write its SI unit, (ii) The electric field components due to a charge inside the cube of side 0.1 m are shown below.




$$E_x = \alpha\, x,$$

where, $\alpha = 500$ N/C - m,

$$E_y = 0, E_z = 0$$

Calculate

(*a*) the flux through the cube and

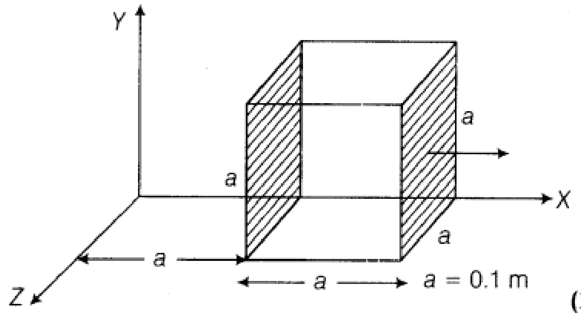
(*b*) the charge inside the cube.

**[All India 2008]**

**Ans.**(i)Electric flux Electric flux over an area in an electric field represents the total number of electric lines of force crossing the area in a direction normal to the plane of the area. The SI unit of electric flux is N-m$^2$/C

(ii) The electric field is directed along +X-axis. Therefore, angle between E and A for left face is 180°, whereas for right face is 0°. The angle between E and A on four non-shaded faces is 90°.

Therefore, flux linked with these four faces is zero.




**(1)**

(a) Total electric flux through the cube ($\phi$)

$$\phi = \phi_L + \phi_R + 0$$

$[\because \phi_{\text{for non-shaded face}} = 0]$

where, flux linked with left face,

$$\phi_L = E_1 \times a^2 \cos 180°$$

where, flux linked with right face,

$$\phi_R = E_2 \times a^2 \cos 0°$$

Total flux passing through cube

$$\therefore \quad \phi = E_1 a^2 \cos 180° + E_2 a^2 \cos 0°$$

$$= -E_1 a^2 + E_2 a^2$$ **(1/2)**

$[\because E_1 = \alpha x = 500 \times 0.1, E_2 = \alpha y = 500 \times 0.2]$

$$= -(500 \times 0.1) \times (0.1)^2 + (500 \times 0.2) \times (0.1)^2$$

$$\phi = -0.5 + 1 = 0.5 \text{ N-m}^2/\text{C}$$

$$\phi = 0.5 \text{ N-m}^2/\text{C}$$ **(1)**

(b) By Gauss' theorem,

$$\phi = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad q = \phi \varepsilon_0$$ **(1)**

$$\therefore \quad q = (0.5) \times 8.85 \times 10^{-12}$$

$$= 4.425 \times 10^{-12} \text{C}$$

Charge inside the cube = $4.425 \times 10^{-12}$C

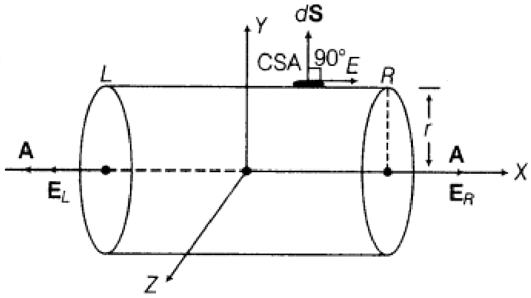
**(1/2)**

**30.** (i) Define electric flux. Write its SI unit.

(ii) A uniform electric field $\mathbf{E} = E_x \hat{\mathbf{i}}$ N/C for $x > 0$ and $\mathbf{E} = -E_x \hat{\mathbf{i}}$ N/C for $x < 0$ are given. A right circular cylinder of length $l$ cm and radius $r$ cm has its centre at the origin and its axis along the X-axis. Find out the net outward flux. Using the Gauss' law, write the expression for the net charge within the cylinder. **[Delhi 2008C]**

**Ans**.(i)Electric flux Electric flux over an area in an electric field represents the total number of electric lines of force crossing the area in a direction normal to the plane of the area. The SI unit of electric flux is N-m$^2$/C

(*ii*) Electric flux linked with curved surface




$$\mathbf{E} \cdot \mathbf{dS} = E\, dS \cos 90° = 0$$ **(1/2)**

According to the problem,

$$\mathbf{E}_R = E_x \hat{\mathbf{i}} \qquad \text{[For } x > 0]$$

$$\mathbf{E}_L = -E_x \hat{\mathbf{i}} \qquad \text{[For } x < 0]$$

∴ Total flux linked with the right circular cylinder

$$\phi = \phi_R + \phi_L$$ **(1/2)**

where, $\phi_R = \mathbf{E}_R \cdot \mathbf{A} = E_R A \cos 0°$

$$= E_x \left(\frac{\pi r^2}{10^4}\right) \qquad [\because A = \pi r^2]$$

$$\phi_L = \mathbf{E}_L \cdot \mathbf{A} = E_L A \cos 0° = E_x \left(\frac{\pi r^2}{10^4}\right)$$

$$[1 \text{ cm} = 10^{-2} \text{ m}]$$

$$\therefore \phi = \phi_R + \phi_L$$

$$= [E_x (\pi r^2) + E_x (\pi r^2)] \times 10^{-4}$$

$$= 2\pi r^2 E_x \times 10^{-4} \text{ N-m}^2\text{/C}$$ **(1)**

By Gauss' theorem, $\phi = \dfrac{q}{\varepsilon_0}$

∴ Net charge $q = \phi \varepsilon_0$

$$= 2\pi r^2 E_x \varepsilon_0 \times 10^{-4} \text{ C}$$ **(1)**

**31.** (*i*) Using Gauss' law, derive an expression for electric field intensity at any point outside a uniformly charged thin spherical shell of radius $R$ and the density $\sigma$ C/m$^2$. Draw the field lines when the charge density of the sphere is

(*a*) positive and

(*b*) negative

(*ii*) A uniformly charged conducting sphere of 2.5 m in diameter has a surface charge density of 100 $\mu$C/m$^2$.

Calculate the

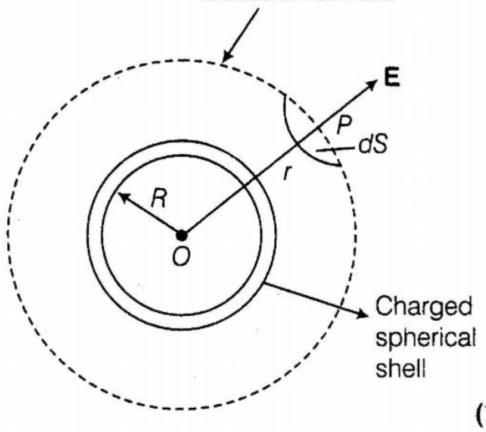
(*a*) charge on the sphere and

(*b*) total electric flux through the sphere. **[Delhi 2008]**

**Ans.** The electric lines of force emerge from the positive charge and comes into the negative charge.

(i)

Electric field due to a uniformly charged thin spherical shell.




**(1)**

(*i*) **When point *P* lies outside the spherical shell**

Suppose that we have to calculate electric field at the point $P$ at a distance $r\,(r > R)$ from its centre. Draw the Gaussian surface through point $P$, so as to enclose the charged spherical shell. The Gaussian surface is a spherical shell of radius $r$ and centre $O$.

Let $\mathbf{E}$ be the electric field at point $P$. Then, the electric flux through area element $d\mathbf{S}$ is given by

$$d\phi = \mathbf{E} \cdot d\mathbf{S} \quad \textbf{(1)}$$

Since, $d\mathbf{S}$ also along normal to the surface.

$$d\phi = E\,dS$$

$\therefore$ Total electric flux through the Gaussian surface is given by

$$\phi = \oint_S E\,dS = E\oint_S dS$$

Now, $\oint dS = 4\pi r^2$

$$\therefore \quad \phi = E \times 4\pi r^2 \quad \text{...(i)}$$

Since, the charge enclosed by the Gaussian surface is $q$. According to Gauss' theorem.

$$\phi = \frac{q}{\varepsilon_0} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we obtain

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \qquad [\text{for } r > R]$$ **(1)**

(*ii*) **When point *P* lies inside the spherical shell**

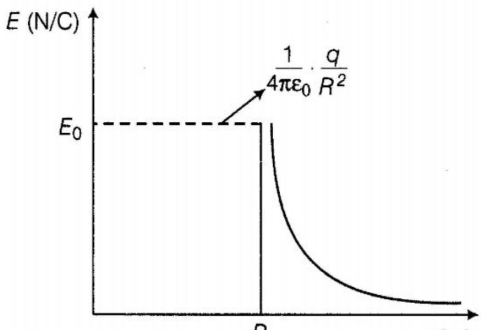
In such a case, the Gaussian surface enclosed no charge.

According to Gauss law,

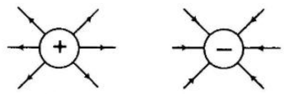
$$E \times 4\pi r^2 = 0$$

*i.e.*, $E = 0$ [for $r < R$]

**Graph showing the variation of electric field as a function of *r*** **(2)**




Electric field lines due to positive and negative charged spherical shell are as given below in figures (a) and (b) I respectively



(a) (b)

(*ii*) (*a*) Charge density, **(1)**

$$\sigma = \frac{q}{4\pi r^2}$$

$$\therefore q = 4\pi r^2 \sigma$$

$$= 4 \times 3.14 \times \left(\frac{2.5}{2}\right)^2 \times 100 \times 10^{-6}$$

$$= 1.9625 \times 10^{-3} \text{C}$$ **(1)**

(*b*) By Gauss' law,

Total electric flux through the sphere

$$\phi = \frac{q}{\varepsilon_0}$$

$$= \frac{1.9625 \times 10^{-3}}{8.85 \times 10^{-12}}$$

$$= 2.2 \times 10^8 \text{ N-m}^2/\text{C}$$ **(1)**